Lecture 4

Introduction to Probability Distributions and Applications in Business

Grace Thomson

Introduction to Probability Distributions

The concept behind probability distribution is very simple, and we need to keep in mind that every probability that we want to assess could be solved if we knew the distribution table.

What is the probability distribution of the sum of two dices rolled simultaneously?

We know that $P(x) = \frac{\text{\# of favorable cases}}{\text{Total \# of cases}}$

We can build a probability distribution table with all the possible results for the sum of the two dices. A probability distribution table tells as about the probability of all the possible ways how an event can happen.

X	P(x)
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Notice that the table doesn't have 0 or 1, because the lowest sum of two dices will be 1+ 1 = 2, right? And the maximum will be…….? 12 of course. The result of the sum is the **variable x** of our distribution table, and **P(x)** is the probability that this sum occurs; it is the probability of the sum being 2, 3, 4, 5, etc.

We know that the probability of rolling a dice and obtaining a given number is 1/6 (classical probability), so the probability of the sum of two dices, given that the event of throwing the dices is independent, will be (1/36) because:

$$P(A + B) = P(A) * P(B) =$$

$$P(A + B) = (1/6) * (1/6) = (1/36)$$

So now, the question is, in how many different ways the sum of 2 dices could yield the result that we want.

Let's think about it. In how many different ways the sum could be 8?

2 + 6 = 8

6+ 2 = 8

3+5 = 8

5+3 = 8

4+4 = 8

Yes, you got it! in 5 different ways. So the probability of having a 8 as a sum is 5*1/36. With this concept we are ready to build our distribution table for all the possible results of sums of dices:

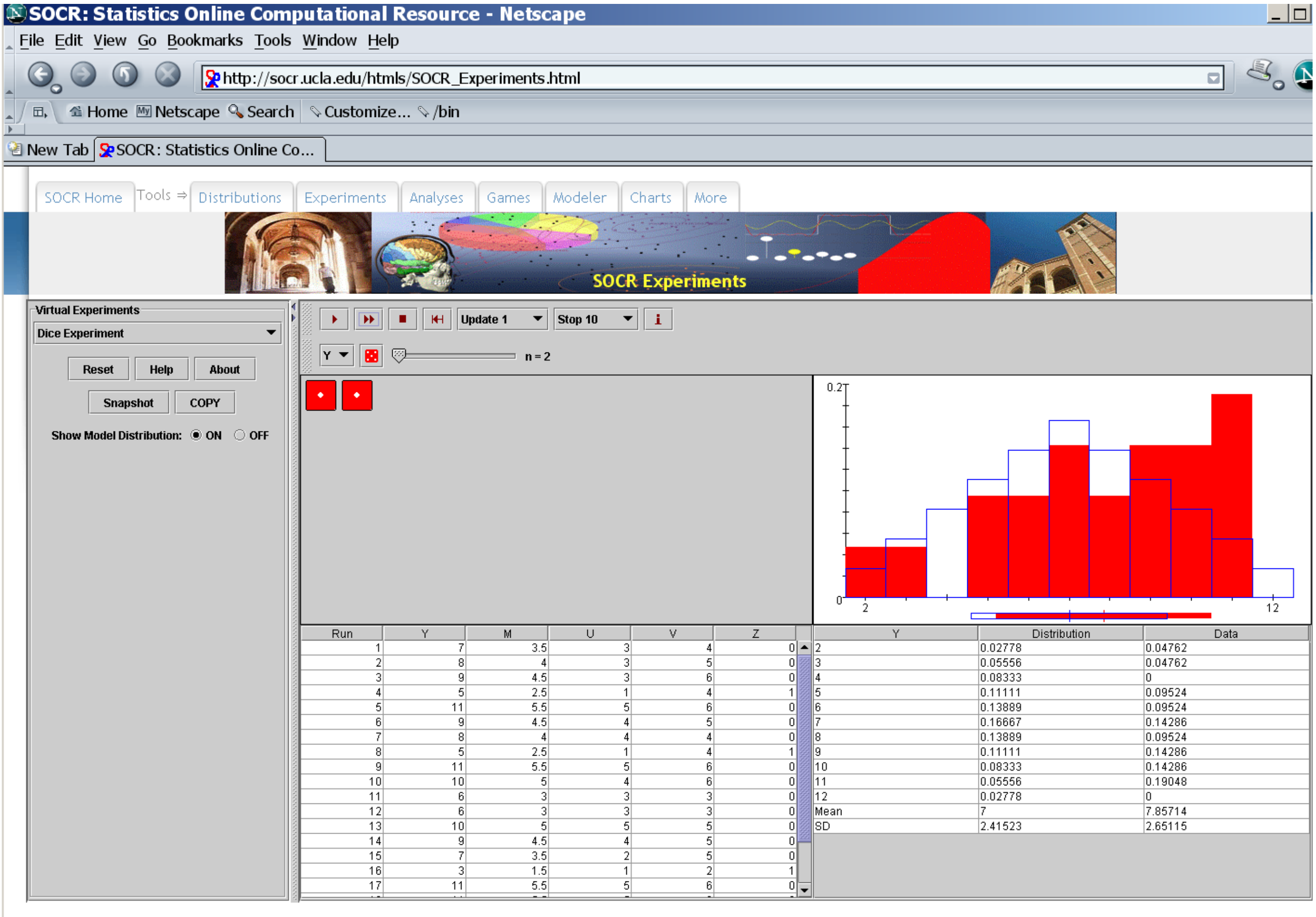
Table 1

Probability distribution for a rolling dice experiment

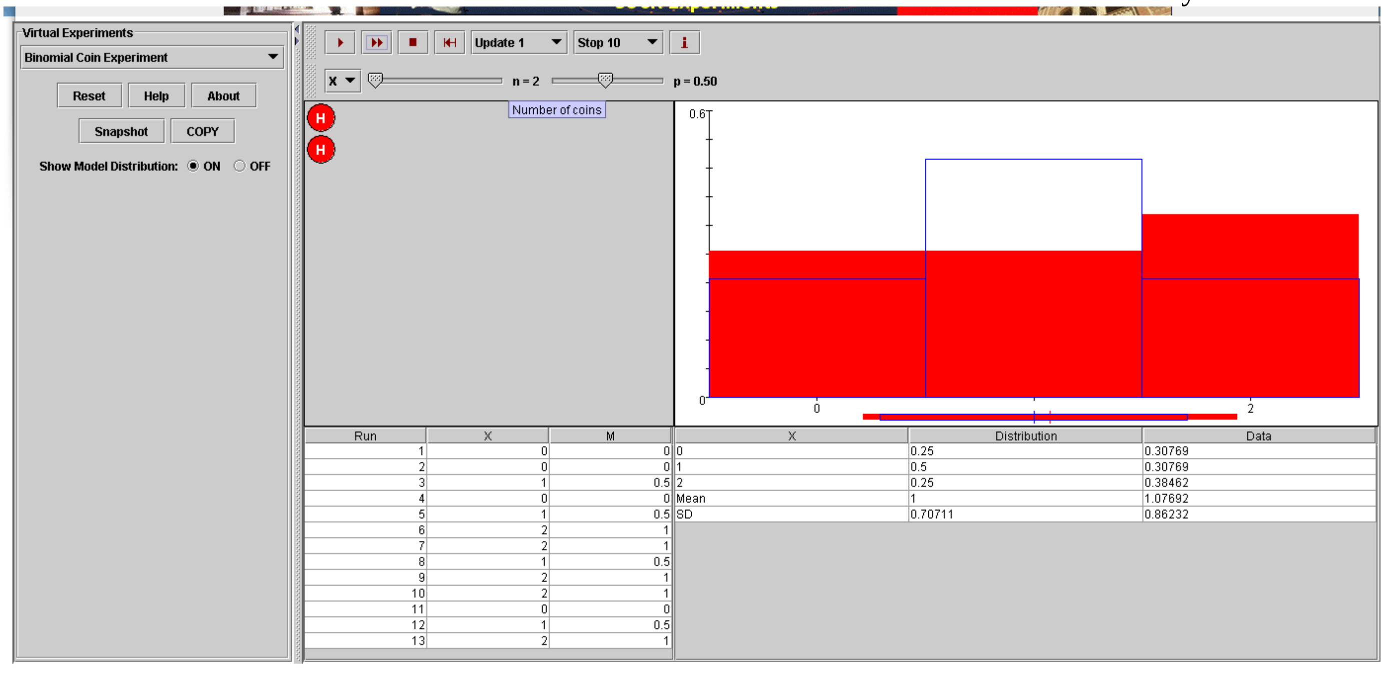
X	# of outcomes	P(x)
2	1 + 1	1/36
3	1+ 2, 2+ 1	2* (1/36)
4	1+ 3, 2+ 2, 3+ 1	3 * (1/36)
5	1+ 4, 2+ 3, 3+ 2, 4+ 1	4* (1/36)
6	1+5, 2+ 4, 3+ 3, 4+2, 5+ 1	5*(1/36)
7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1	6*(1/36)
8	2+6, 3+5, 4+4, 5+3, 6+2	5*(1/36)
9	3+6, 4+5, 5+4, 6+3	4* (1/36)
10	4+6, 5+5, 6+4	3 * (1/36)
11	5+6, 6+5	2* (1/36)
12	6+6	1/36

This is called a MULTINOMIAL DISTRIBUTION. You can imagine that for larger experiments, specialized formulas and software assist us in counting the number of repetitions.

See this SOCR Experiment (http://socr.ucla.edu/htmls/SOCR_Experiments.html), **Dice Experiment**, choose N=2 and Run the experiment several times. Notice the theoretical (blue) and empirical distributions (red) in the graphs and tables!



Similarly try the **Binomial Coin Experiment** for N=2 coins. See if your Distribution calculations match the ones reported in the applet.



Virtual Experiments
Binomial Coin Experiment
Reset
Help
About
Snapshot
COPY
Show Model Distribution: ON OFF
Update 1
Stop 10
n = 2
p = 0.50
Number of coins
H
H
0.6
0
0
2
Run
X
M
1 0 0
2 0 0
3 1 0.5
4 0 0
5 1 0.5
6 2 1
7 2 1
8 1 0.5
9 2 1
10 2 1
11 0 0
12 1 0.5
13 2 1
X
Distribution
Data
0 0.25 0.30769
1 0.5 0.30769
2 0.25 0.38462
Mean 1 1.07692
SD 0.70711 0.86232

How to compute the mean and the standard deviation of a Distribution of Probabilities for a Discrete Variable

Let's say that we have a probability distribution table for the potential earnings of a project A is as follows:

Table 2

Probability Distribution of Earnings of Project A

Earnings	P(x)
100,000	0.1
200,000	0.2
300,000	0.3
350,000	0.2
400,000	0.2

We can compute the mean and the standard deviation of this table, using the following formulas. The Mean of a probability distribution is also called "Expected value".

Expected value: It's the average value when an experiment that is repeated over the long run.

Expected value for 1 variable

$$E(x) = \Sigma\, x\, P(x)$$

Expected value for 2 variables

$$\mathbf{E(x + y) = E(x) + E(y)}$$

"expected value of the sum of variables"

Standard Deviation

Similarly to what we learned before, it measures the spread in the values of a random variable.

$$\sigma_x = \sqrt{\Sigma[x - E(x)]^2 * P(x)}$$

LEARNING TEAM ACTIVITY

Compute the mean and the standard deviation for the table above.

MEAN: $290,000
STANDARD DEVIATION: $91,652

SOLUTION- DELETE FROM STUDENT'S VERSION

Earnings	P(x)	x P(x)	x-Ex	(x-Ex)^2	(x-Ex)^2 *(Px)
100,000	**0.1**	$ 10,000	$ (190,000)	$ 36,100,000,000	$ 3,610,000,000
200,000	**0.2**	$ 40,000	$ (90,000)	$ 8,100,000,000	$ 1,620,000,000
300,000	**0.3**	$ 90,000	$ 10,000	$ 100,000,000	$ 30,000,000
350,000	**0.2**	$ 70,000	$ 60,000	$ 3,600,000,000	$ 720,000,000
400,000	**0.2**	$ 80,000	$ 110,000	$ 12,100,000,000	$ 2,420,000,000
		$ 290,000			**$ 8,400,000,000**

Expected value= SUM[x P(x)] E(x)= $ 290,000

Standard deviation = $\sigma_x = \sqrt{\Sigma[x - E(x)]^2 * P(x)}$ Std. dev = $ 91,652

Basic Application of Probability Distributions to Game Theory

Now let's apply this knowledge to work an interesting concept of Game Theory, the return to risk ratio. We will address more topics about Game Theory in the last chapters of the course.

Return to risk ratio

This ratio is a computed as a quotient between the mean and the standard deviation. An interesting application of this concept is its use to compare investment options. If you have two projects that you are interested in promoting, you can certainly make better business decisions if you compute this ratio.

$$r = \frac{\mu}{\sigma}$$

Return to risk ratio:

Divide mean by standard deviation,

The higher the number, the better the option.

LEARNING TEAM ACTIVITY

Project 1 has an expected value of earnings of $290,000 for the next 5 years, while Project 2 has an expected value of earnings of $300,000. The standard deviation of earnings for project 1 is $91,652, while the standard deviation of earnings for project 2 is $120,000.

Compute the RETURN TO RISK ratio and explain what project will you select?