Hidden Markov Model for High Frequency Data

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What are HMMs?

A Hidden Markov model (HMM) is a stochastic signal model which has three assumptions:

- The observation at time t, O_t, was generated by some process whose state, S_t, is **hidden**.
- The hidden process satisfies the first-order Markov property: given S_{t-1}, S_t is independent of S_i for any $i < t - 1$.
- The hidden state variable is discrete.

History of HMMs

- Introduced in 1966 by Baum and Petrie
- Baum and his colleagues published HMM training for a single observation, 1970
- Levonson, Rabiner, and Sondhi presented HMM training for multiple independent observations, 1983
- Li, Parizeau, and Plamondo introduced HMM traning for multiple observations, 2000

Some applications of HMMs

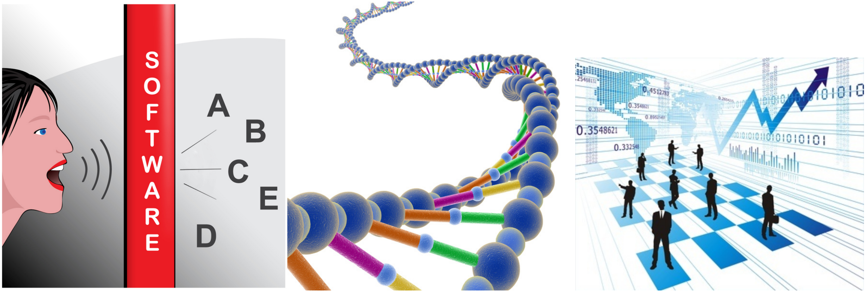


Figure : 1. Speech recognition 2. Bioinformatics 3. Finance

Elements of HMM

- Observation data, $O = (O_t)$, $t = 1, .., T$
- Hidden states, $S = (S_i), i = 1, 2, ..., N$
- Hidden state sequence: $Q = (q_t)$, $t = 1, ..., T$
- Transition matrix A

$$a_{ij} = P(q_t = S_j | q_{t-1} = S_i), \; i, j = 1, 2, ..., N$$

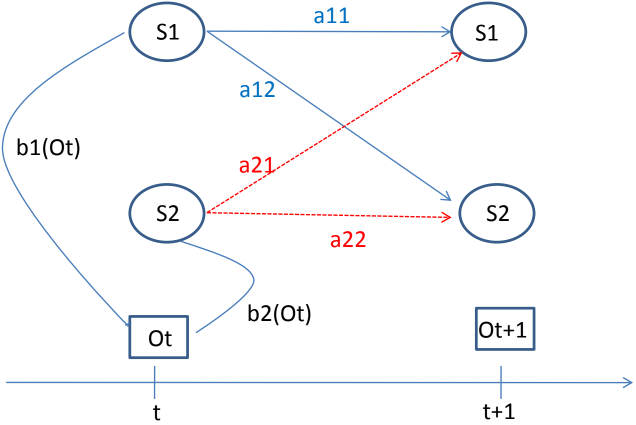
- Observation symbols per state, $V = (v_k), k = 1, 2, ..., M$
- The observation probability

$$B : b_i(k) = P(O_t = v_k | q_t = S_i), i = 1, 2, ..., N; k = 1, 2, ..., M$$

- Initial probabilities, vector p, of being in state S_i at $t = 1$

$$p_i = P(q_1 = S_i), \; i = 1, 2, ..., N$$

Hidden Markov Model



Parameters of HMM: $\lambda = \{A, B, p\}$

Three problems and corresponding solutions for HMMs

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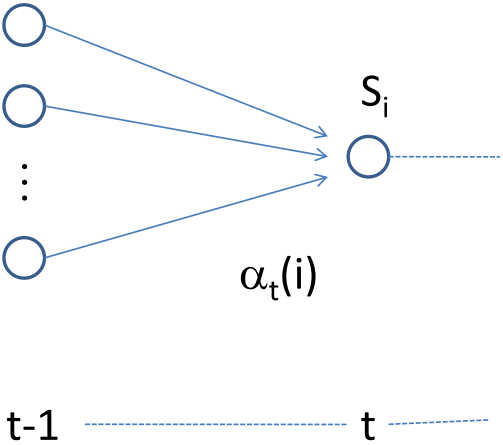
   Baum-Welch algorithm

Forward Algorithm

Define the joint probability $\alpha_t(i) = P(O_1, O_2, ..., O_t, q_t = S_i|\lambda)$



Forward algorithm

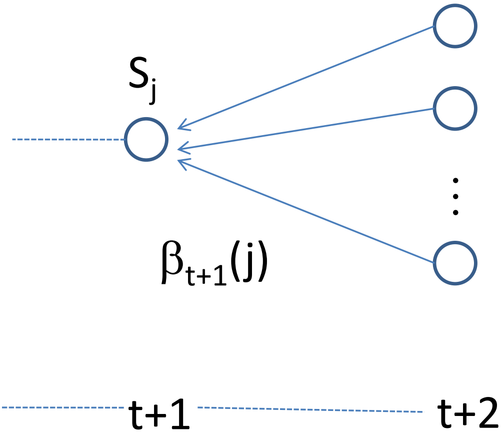
- Initialization, $\alpha_1(i) = p_i b_i(O_1)$ for $i = 1, ..., N$
- For $t = 2, 3, ..., T$, for $j = 1, ..., N$

$$\alpha_t(j) = \left[\sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} \right] b_j(O_t),$$

- $P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)$

Backward Algorithm

Define the conditional probability
$\beta_t(j) = P(O_{t+1}, O_{t+2}, .., O_T | q_t = S_j, \lambda)$, for $j = 1, ..., N$



Backward Algorithm

Algorithm

- Initialization, $\beta_T(i) = 1$ for $i = 1, ..., N$
- For $t = T-1, T-2, ..., 1$, for $i = 1, ..., N$

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)$$

- $P(O|\lambda) = \sum_{i=1}^{N} p_i b_i(O_1) \beta_1(i)$

Choose economics indicators

1. Inflation (CPI)
2. Credit Index
3. Yield Curve
4. Commodity
5. Dow Jones Industrial Average

Training and Predicting Process

Using the variables above:

- Use HMM for single and multiple observation data with normal distributions.
- Calibrate Markov-switching model parameters using Baum-Welch algorithm
- Define state or regime 2 with lower *mean*/*variance*
- Use the obtained parameters to predict the corresponding states (regimes), predict the upcoming regime.

Results

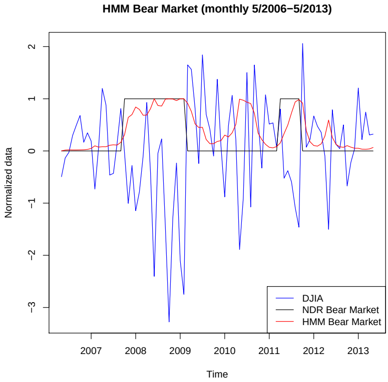


Figure : Dow Jones observations vs probabilities of being in the bear market

Results

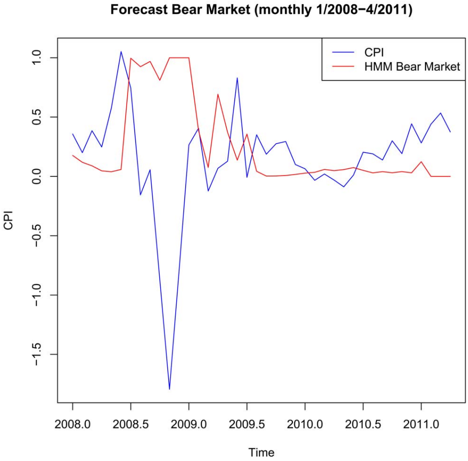


Figure : Forecast bear market using CPI indicator

Results

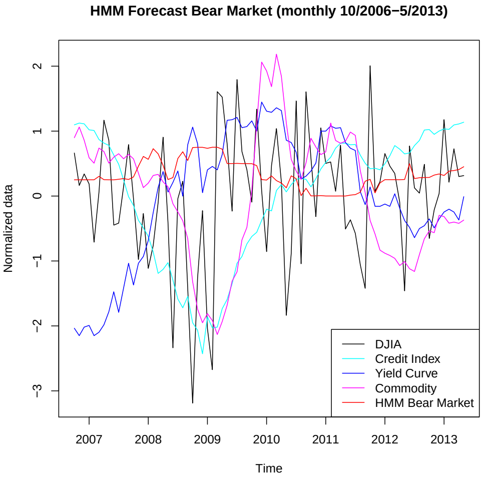


Figure : Forecast bear market using multiple observations

- *S&P* 500, a stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. Monthly percentage changes from February 1947 through June 2013.
- SPY
- GOOG
- FORD
- AAPL
- GE

Training and Predicting Process

Using the variables above:

- Use HMM for single and multiple observation data with normal distributions.
- Calibrate Markov-switching model parameters using Baum-Welch algorithm
- Use the obtained parameters to predict stock prices for the next trading period.

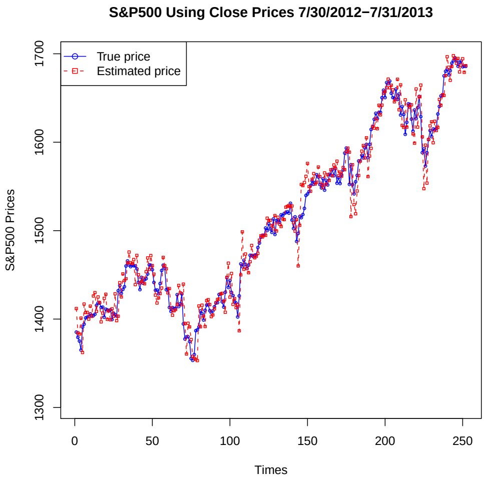


Figure : Forecast $S\&P500$ close prices using single observation

Results

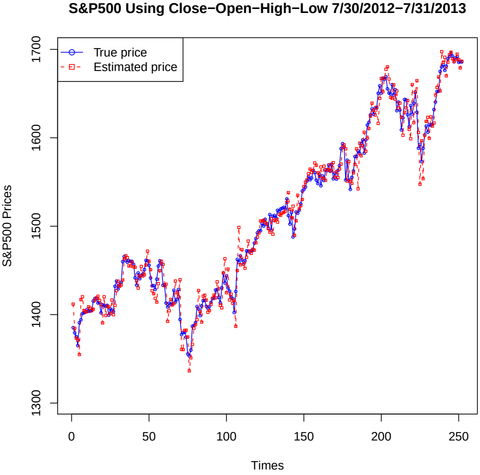


Figure : Forecast $S\&P500$ closing prices using multiple observations (open-close-high-low)

Results

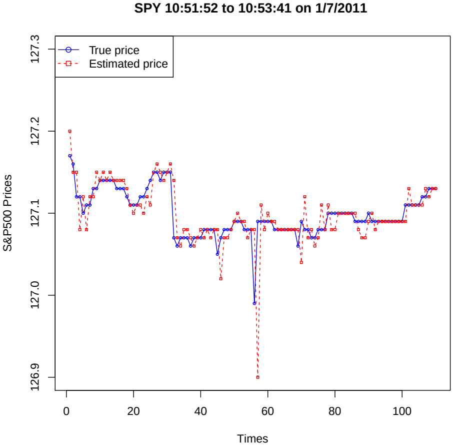


Figure : Forecast *SPY* bid price in tick by tick

Can we use HMMs to make money?

Symbol	Initial Investment ($)	Earning ($)	Earning %
SPY	9,000.00	2050.66	22.79
GOOG	30,000.00	29,036.4	96.79
FORD	250.00	10.10	4.04
AAPL	950.00	19.06	2.01
GE	1,700.00	490.00	28.82
TOTAL	41,900.00	31,606.22	75.43

Table : One year daily stock trading portfolio from December 2012 to December 2013

Thank you!

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